By Martin Gutekunst, Observatory Eberfing, data also from Gregor Krannich and Carsten Ziolek

#### **Question:**

Which exposure time leads to the best time precision regarding time? (for events with deep dip delta mag < 3)

**Basics for Signal statistics** 

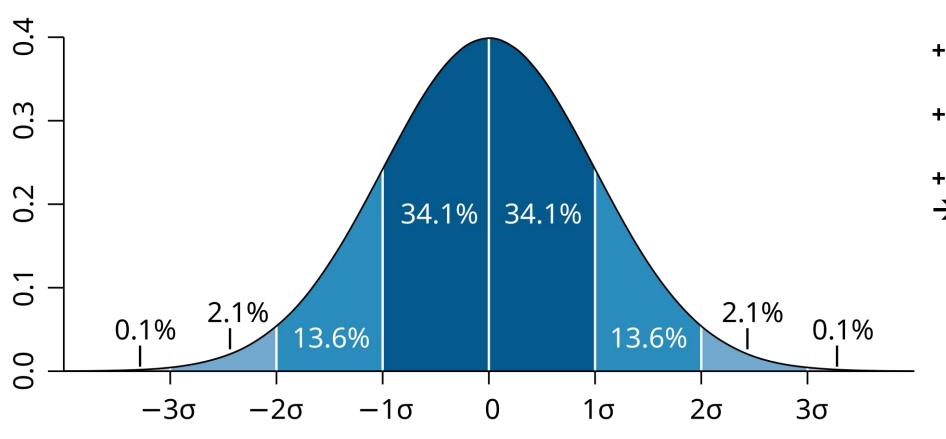
Two main components for Noise:

- Noise due to seeing: linear function of integration time (personal observation)
- Noise due to photons:
  - proportional to Square root of photons
  - → so proportion to square root of integration time

The question which error dominates?

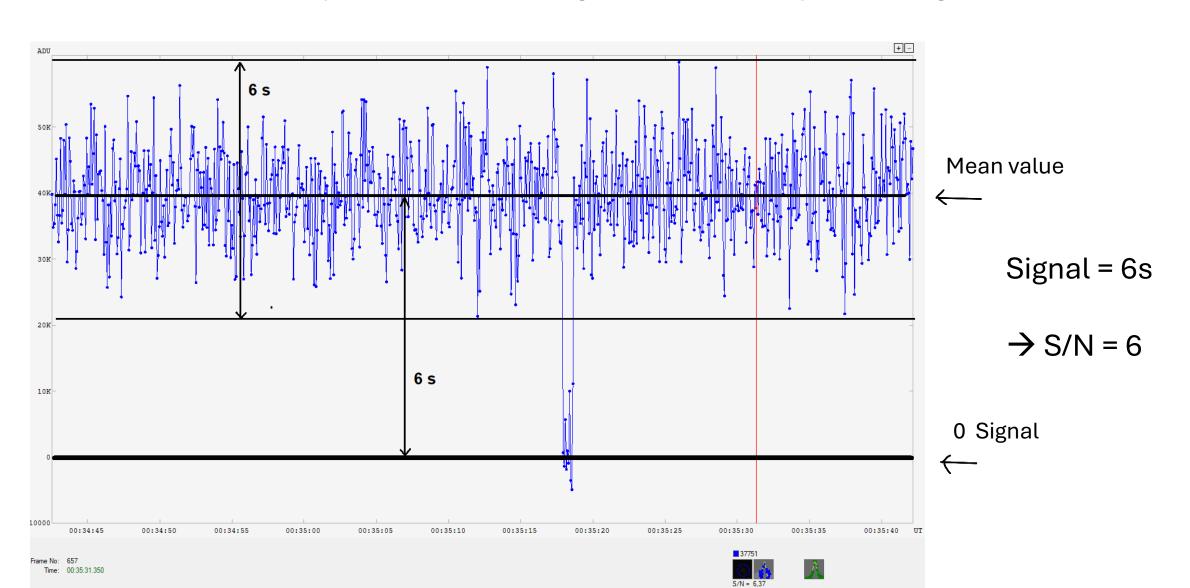
- → we regard 2 options: → noise limited by seeing
  - → noise limited by photons

# Basics for Noise consideration: stochastic error can be described with the Gaussian Normal distribution

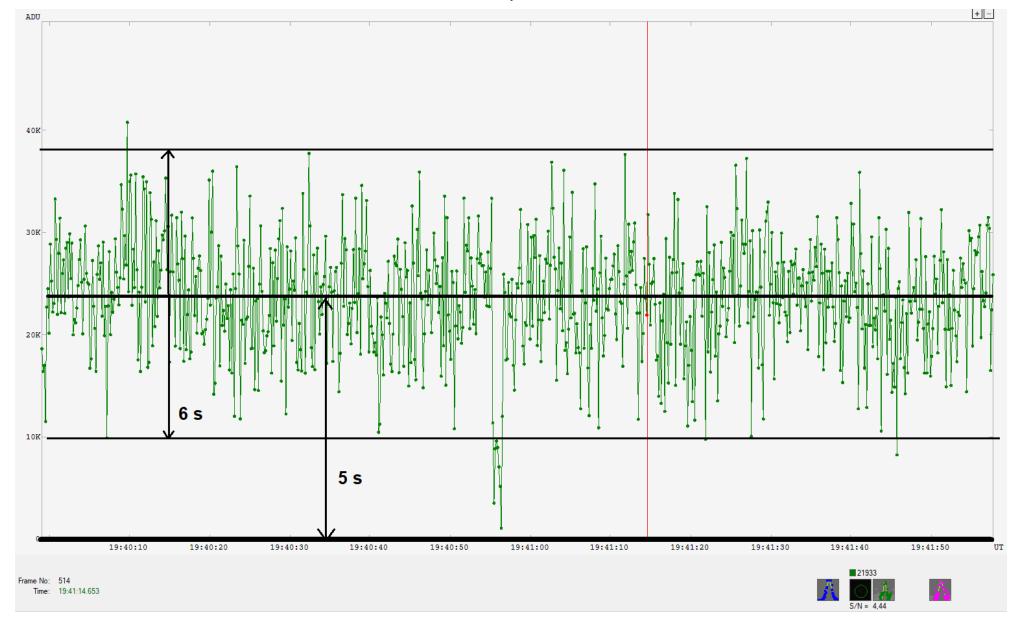


- +/- 1s in Interval 68,3 %
- +/- 2s in Interval 95,4 %
- +/- 3s in Interval 99,7%
- → 3 of 1000 datapoint are outside Interval

A simple method for estimating the S/N ratio: Example SN 6s range



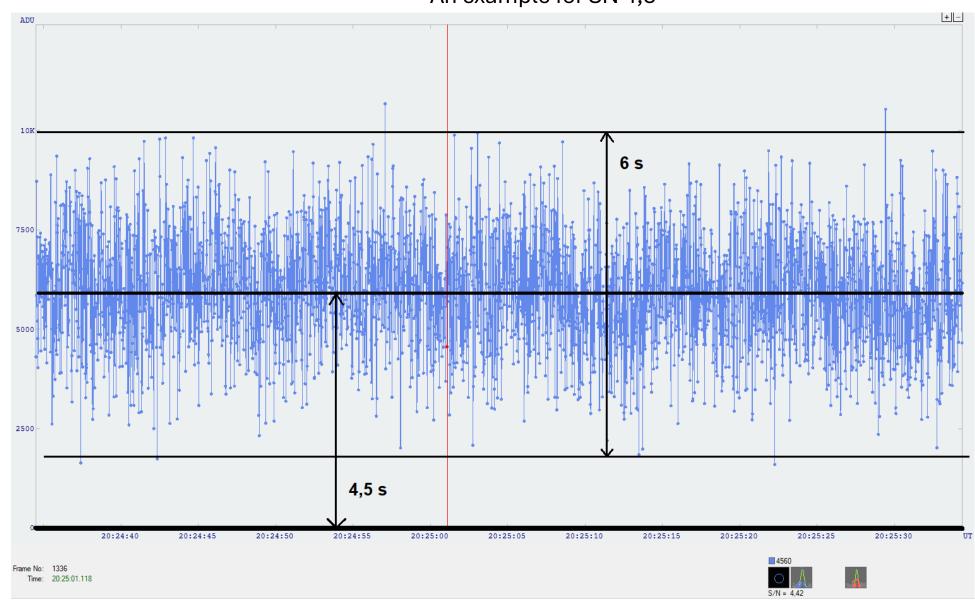
An example for SN 5



Signal = 5s

 $\rightarrow$  S/N = 5

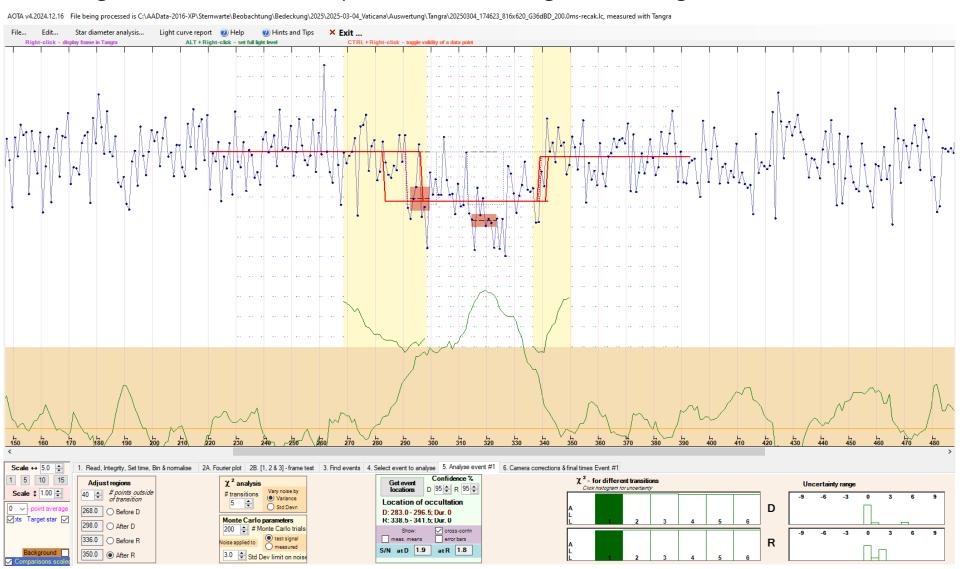




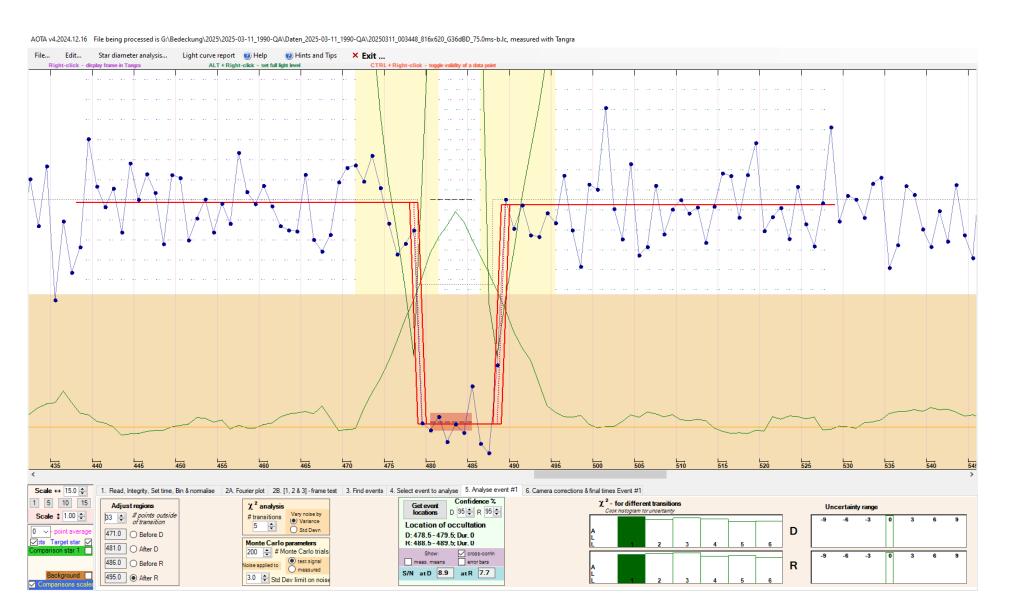
Signal = 4,5s

 $\rightarrow$  S/N = 4,5

#### An Integration time as short as possible with low Signal Noise e.g. 2 ?



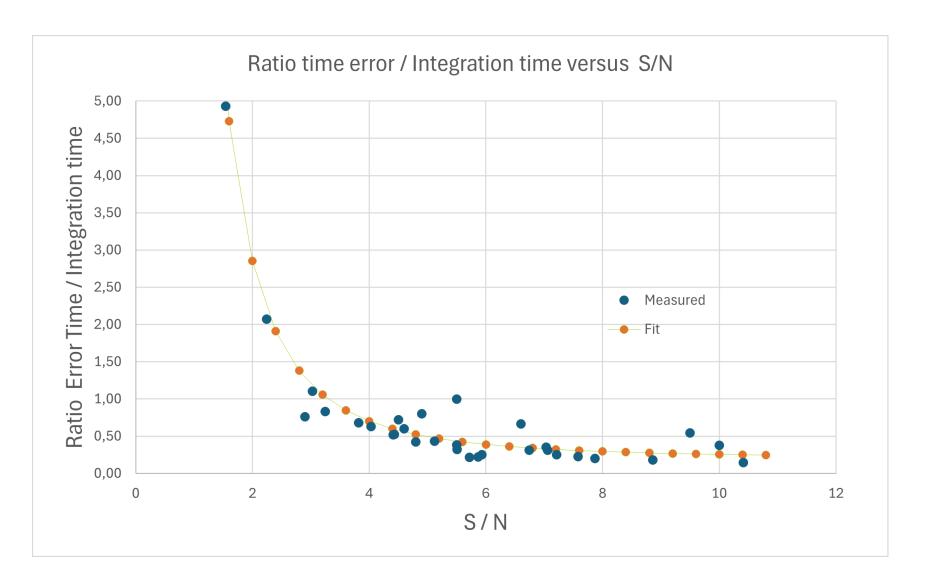
A longer integration time to have better S/N Raio and better significance to detect. But Time resolution ???



#### **Observation:**

The time precision is different than the integration time

|                    |             |         |                      | Precision/  |  |
|--------------------|-------------|---------|----------------------|-------------|--|
|                    | Integration |         | <b>Precission in</b> | Integration |  |
| Asteorid           | Time        | S/N dip | ms                   | Time        |  |
| Zuber              | 15          | 10,73   | 3,3                  | 0,220       |  |
| 2024-12-31_Handley | 40          | 8,03    | 9                    | 0,225       |  |
| Zeuxo              | 20          | 6,33    | 3,7                  | 0,185       |  |
| 1990 SF            | 50          | 5,75    | 17                   | 0,340       |  |
| Thymbrraeus        | 60          | 5,6     | 24                   | 0,400       |  |
| 1990-QA            | 75          | 4,72    | 28,9                 | 0,385       |  |
| Alessiosquillon    | 33          | 4,53    | 17                   | 0,515       |  |
| Odysseus           | 500         | 4,2     | 442                  | 0,884       |  |
| 2003 YM9           | 40          | 4,11    | 18                   | 0,450       |  |
| Lowell             | 150         | 4,1     | 99                   | 0,660       |  |
| Pobeda             | 300         | 3,92    | 141                  | 0,470       |  |
| 2002 TY52          | 175         | 3,9     | 130                  | 0,743       |  |
| Fiducia            | 100         | 3,8     | 87                   | 0,870       |  |
| Eucharis           | 40          | 3,72    | 26                   | 0,650       |  |
| Carolyn            | 200         | 3,42    | 125                  | 0,625       |  |
| Rhone              | 200         | 3,4     | 130                  | 0,650       |  |
| Clementina         | 500         | 3,33    | 436                  | 0,872       |  |
| Ornamenta          | 500         | 3,33    | 374                  | 0,748       |  |
| 2003-PV20          | 35          | 3,29    | 25                   | 0,714       |  |
| Maresjev           | 75          | 3,22    | 72                   | 0,960       |  |
| Werkhoven          | 100         | 3,18    | 78                   | 0,780       |  |



We observe:

with increasing S/N ratio the Ratio Error-Time regarding integration time decreases

For SN < 3 Error time is bigger than Integration time

First hint: Avoid S/N < 3

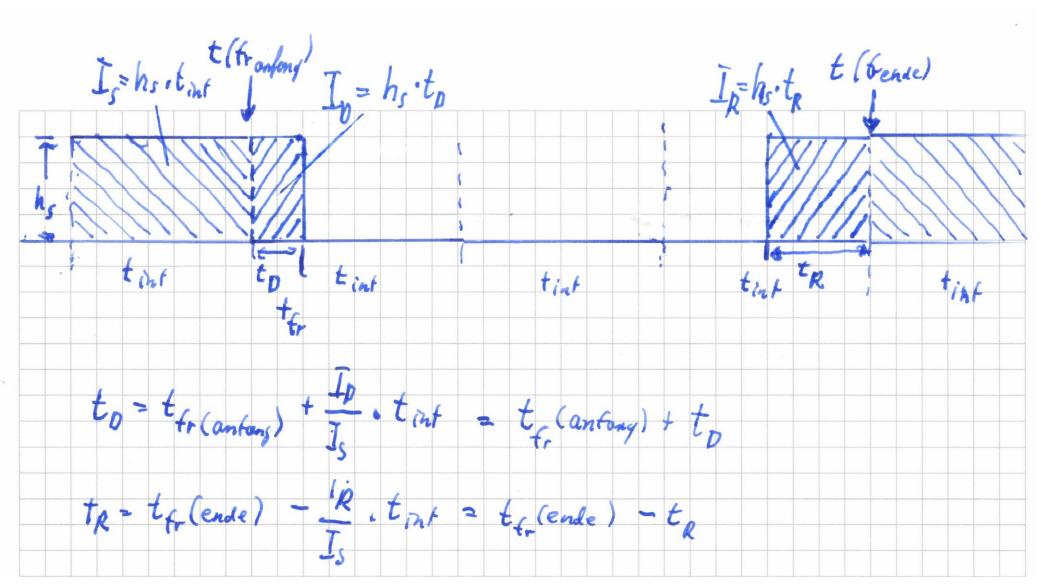
Question: Can a higher S/N ratio also achieve a smaller time error?

Let's take a closer look at this.

To simplify matters, let us consider the case where for the dip debth delta\_Mag\_ dip > 3 S.

The fraction of the frame interval for the D- or R-time are measured by taking the Intensity ratios of the frame when the signal disappears or recovers regarding the signal of the frames when the star is not occulted.

See following figure:



Td = Id/Is \* Tintegr

Martin Gutekunst

#### A short theory with following restrictions

- 1. Signal of asteroid is negligible: dips > 3mag and
- 2. Since Basis of dip is  $0 \rightarrow SN(dip) = SN = Is / \delta Is$
- 3. For a given Integration time the Noise is proportional Square root of signal  $\rightarrow$  ( $\delta$  Id/ $\delta$ Is)<sup>2</sup> = Id / Is (Noise in comparable from frame to frame)

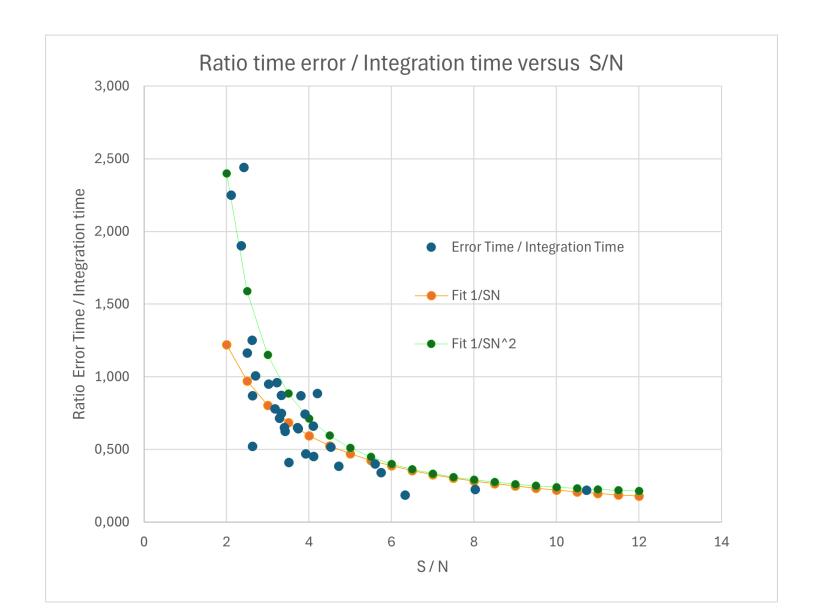
Using quadratic error propagation and we obtain (see add):

$$\delta$$
 (td / Tintegr) = (0,5 + Id/Is) / SN)

$$\rightarrow$$
 for Id << Is:  $\delta$  (td / Tintegr) = 0,5 / SN

$$\rightarrow$$
 for Id = Is:  $\delta$  (td / Tintegr) = 1,5 / SN

$$\rightarrow$$
  $\delta$  (td / Tintegr) ~ 1 / SN:

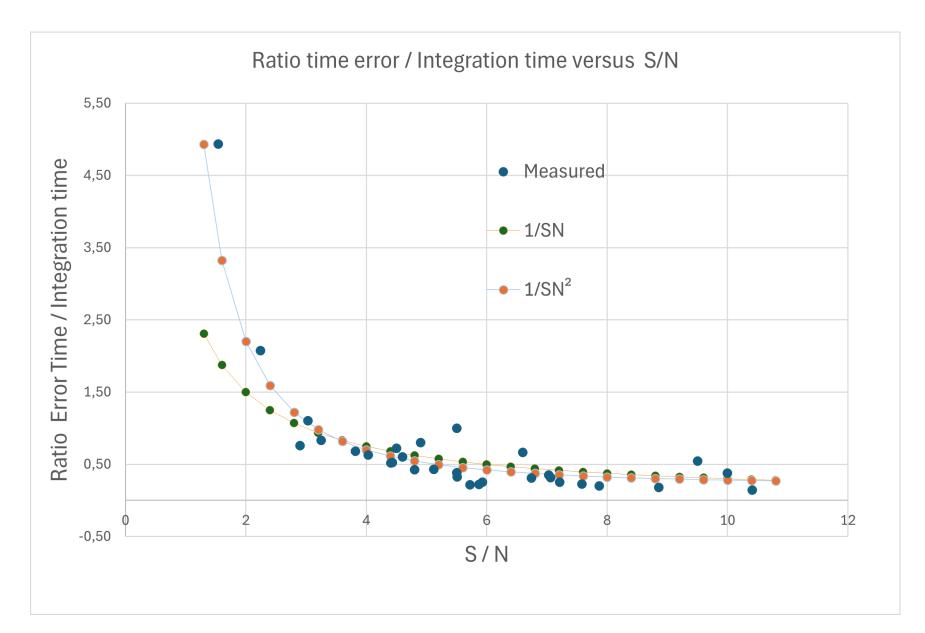


Data by Martin Gutekunst

January 2024 to August 2025

For SN > 3 error Td/Integrationtime ~ 1 / SN

For SN < 3 error Td/Integrationtime ~ 1 / SN<sup>2</sup>

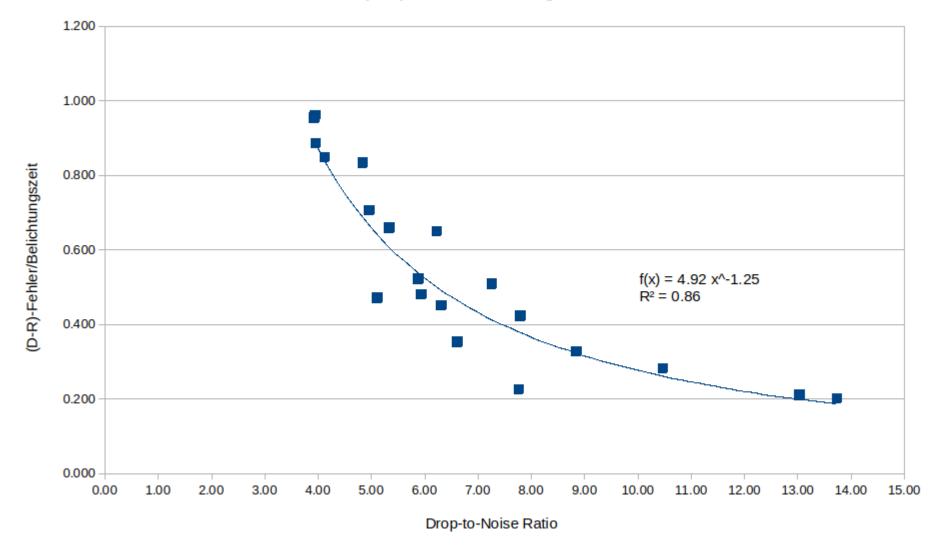


Data by Gregor Krannich

For SN > 3 error Td/Integrationtime ~ 1 / SN

For SN < 3 error Td/Integrationtime ~ 1 / SN<sup>2</sup>





Data by Carsten Ziolek

#### **Overall Result**

For data with SN(Dip) > 3, the ratio (Error(t) / integration time with respect to SN(Dip) follows a power function of the form:

Ratio(Error(t)/Tintegr) =  $a * SN^{-1} + c$ 

Overall, it can be seen that below S/N = 3 the relative error increases exceptional (Power factor at least -2) and the achieved time precision is worser than the integration time

 $\rightarrow$  Avoid a S/N < 3!

- → the ratio Error(t) / Integration time = 0,2 seems to be a limit
- → There might be maximal a factor 5 more resolution regarding the integration time

#### What means that now to the absolute time error?

The absolute time error calculates to:

- $\rightarrow$   $\delta$  (td) ~ 1/SN\* Integration time
- → We can consider two different scenarios:

#### **Scenario 1:**

Seeing dominates the noise: SN is proportional Integration time:

- $\rightarrow$   $\delta$  (td) ~ 1/ Integration time \* Integration time  $\rightarrow$   $\delta$  (td) = constant
- → Integration time doesn't matter.

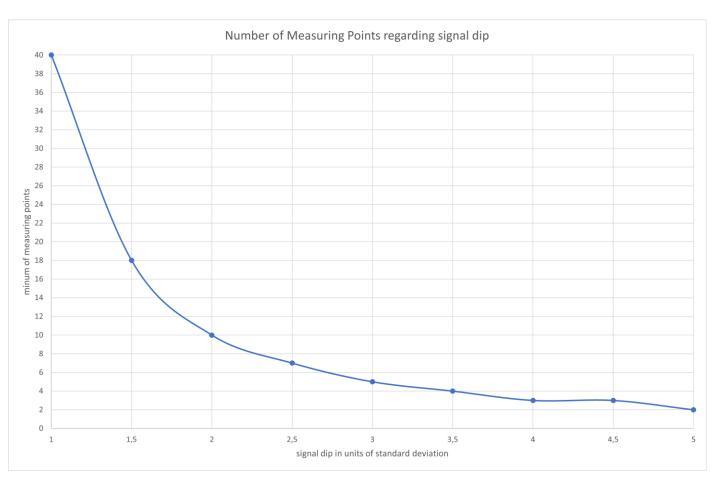
#### Scenario 2:

SN is proportional to the square root (integration time): Photon noise limited Photon noise dominates due to its SN dependence with Square root (Tintegration)

- $\rightarrow$   $\delta$  (td) ~1/ Square root (Integration time) \* Integration time  $\rightarrow$   $\delta$  (td) ~ Square root (integration time)
- → The short the integration time the better

#### The short the integration time the better --- is that really right ??

#### The second parameter is the significance of the dip value



The diagram was calculated by Pyote

So, for short events e.g. only 2 to 3 datapoints per dip we have to chose a Signal / Noise level of at least 4 to 5

Pyote algo accepts a positive occultation for 5 data points with a signal of S/N = 3 3 data points with a Signal of S/N = 4 2 data points with a Signal of S/N = 5

With an S/N of 3 you need 5 data point on the dip ground!

What time accuracy we are losing at a Signal with SN 5 compared to S/N of 3?

Scenario 1: SN rises linear with exposure time (limited by seeing)

- → detection limit of dip width is factor 0,63 smaller for SN 5 regarding to SN 3
- → time precision of SN5 is the same regarding SN3
- → Integration time for SN 5 should be chosen regarding overall data robustness combined with sensitivity and dip width behaviour

| SN | Integration Time rel to SN(3) | time precision rel to SN(3) | Min datapoint for dip | Min dip width in time rel to SN (3) |
|----|-------------------------------|-----------------------------|-----------------------|-------------------------------------|
| 3  | 1                             | 1                           | 5                     | 5                                   |
| 4  | 1,33                          | 1                           | 3                     | 4,0                                 |
| 5  | 1,67                          | 1                           | 2                     | 3,14                                |

What time accuracy we are losing at a Signal with SN 5 compared to S/N of 3?

Scenario 2: SN rises proportional to the square root of the exposure time (photon statistic is the dominant factor)

- → detection limit of dip for SN 3 10% better as for SN of 5
- → time precision of SN5 is Factor 1,66 less regarding SN3
- → SN 3 optimal

| SN | Integration Time rel to SN(3) | time precision rel to SN(3) | Min datapoint for dip | Min dip width in time rel to SN (3) |
|----|-------------------------------|-----------------------------|-----------------------|-------------------------------------|
| 3  | 1                             | 1                           | 5                     | 5                                   |
| 4  | 1,77                          | 1,33                        | 3                     | 5,3                                 |
| 5  | 2,78                          | 1,67                        | 2                     | 5,5                                 |

#### **Conclusion:**

Since my personal experience shows a linear function regarding the increase of the SN in respect to the integration time:

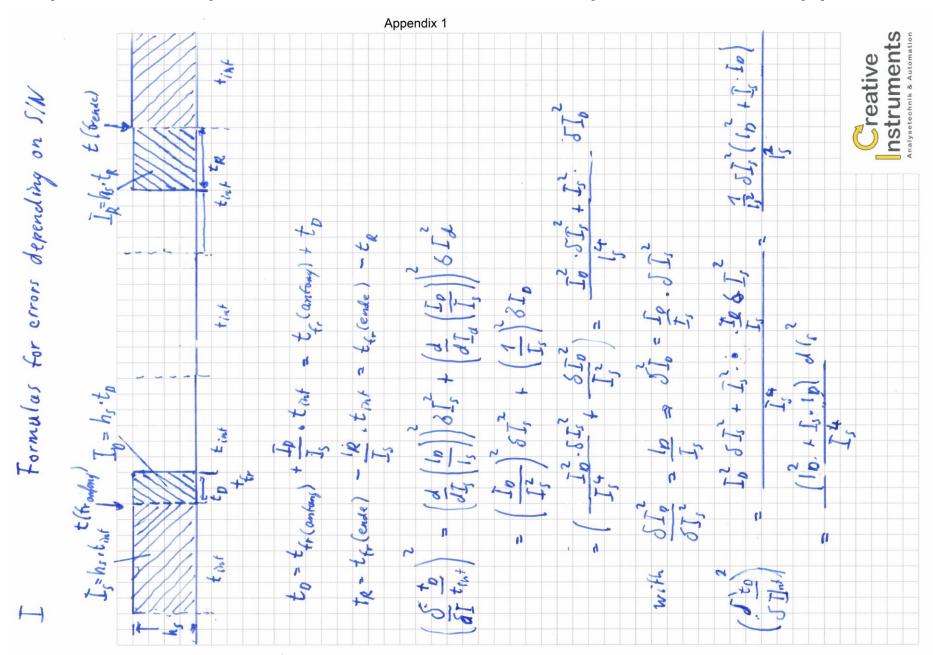
I propose to follow Scenario 1

Record data with an S/N of least 4–5:

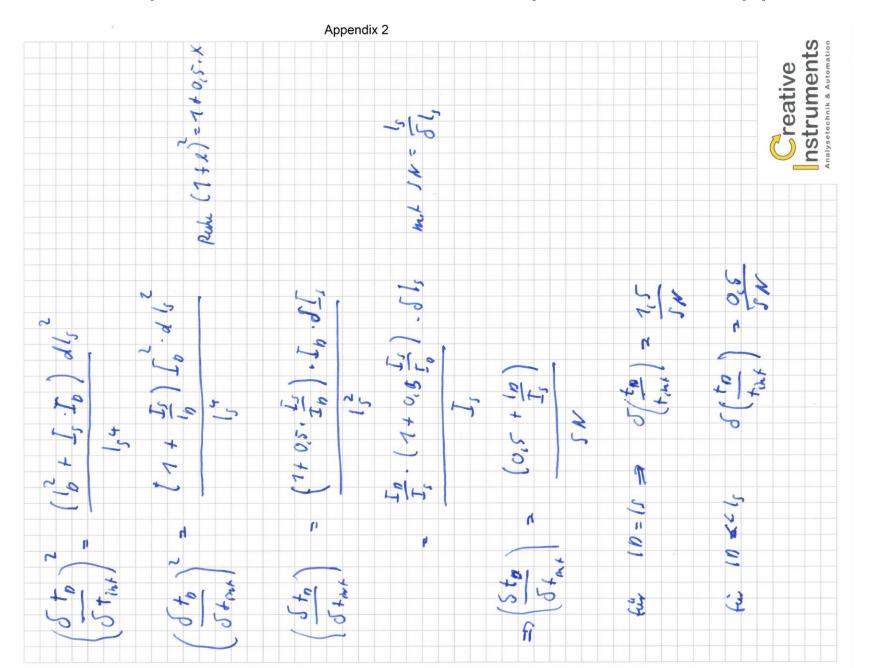
- This ensures that You the star is always clearly above the background and the light curve can be extracted more easily (no lost frames when the star disappears in the background at 3s).
- Precision is the same for SN 5 in respect to SN 3
- a more robust detection of positiv events with few datapoints (dips with 30% less occultation time are still detected)

# Thank you for your attention

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Formulas for errors depending on SIN

13

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